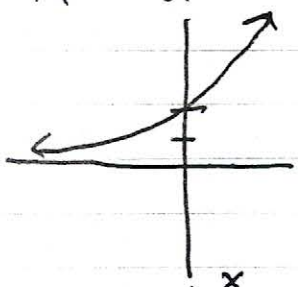
Pg 302
6, 10, 14-21, 24

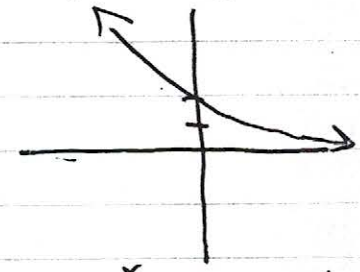
6.1 Exponential Functions

Exponential Growth



$y = a \cdot b^x$ $\ast\ b > 1\ \ast$
d = {all real numbers}
r = {$y \geq 0$}
intercept $(0, a)$
End behavior
as $x \to -\infty$, $y \to 0$
as $x \to \infty$, $y \to \infty$

Exponential Decay



$y = a \cdot b^x$ $\ast\ 0 < b < 1\ \ast$
d = all real numbers
r = $y \geq 0$
intercept $(0, a)$
End behavior
as $x \to -\infty$, $y \to \infty$
as $x \to \infty$, $y \to 0$

ex) Is the following function growth? or Decay?

a) $f(x) = 2^x$
growth

b) $g(x) = 5\left(\frac{1}{2}\right)^x$
Decay $\frac{1}{2}$

Exponential Growth model

$y = a(1+r)^t$
a = initial Amount
r = rate as a decimal
t = time

Exponential Decay

$y = a(1-r)^t$

ex) A new car costs \$25,000. The value of the car decreases 15% a year. How much is the car worth after 4 years?

Decay
$y = a(1-r)^t$
$a = 25{,}000$
$r = 15\% = .15$
$t = 4$

$y = 25{,}000(1-.15)^4$

$y = \$13{,}050.16$

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7-11, 16-24

6.2 Exponential Models

Compound Interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A = Future Value
P = Present Value
r = rate as a decimal
n = number of times compounded per year
t = time in years

Compound Continuously

$$A = Pe^{rt}$$

A = Future Value
P = Present Value
e = natural base (button on calculator)
r = rate as a decimal
t = time in years

ex) You invest $5000 in an account that pays 4% interest. How much will be in the account after 3 years if the interest is compounded monthly?

A = ?
P = 5000
r = 4% = .04
n = 12
t = 3

$$A = 5000\left(1 + \frac{.04}{12}\right)^{(12 \cdot 3)}$$

A = 5,636.36

over →

b) Compounded Continuously

$A = ?$
$P = 5000$
$r = 4\% = .04$
$t = 3$

$$A = 5000e^{(.04 \cdot 3)} = \$5,637.48$$

ex) write an exponential model given two points

a) (15, 10) (16, 14)

$$y = a \cdot b^x$$

$$b = \frac{14}{10} = 1.4$$

$$y = a(1.4)^x$$

plug in a point to find a

$$\frac{10}{1.4^{15}} = \frac{a(1.4)^{15}}{1.4^{15}}$$

$$a = .06$$

$$y = .06(1.4)^x$$

Pg 319
23-49 odd
(maybe all)

6.3 Logarithms

Log Form $\log_b x = y$

Exponent Form $y = b^x$

ex) Convert from exponent form to log form

a) $3^4 = 81$ b) $7^3 = 343$

$\log_3 81 = 4$ $\log_7 343 = 3$

ex) Convert from log form to exponent form

a) $\log_{10} 1000 = 3$ b) $\log_4 16 = 2$

$10^3 = 1000$ $4^2 = 16$

☆ Change of base ☆

$$\log_x y = \frac{\log y}{\log x}$$

ex) Evaluate

a) $\log_5 125 = \frac{\log 125}{\log 5} = 3$

b) $\log_3 (\frac{1}{9}) = \frac{\log (1/9)}{\log (3)} = -2$

c) $\log 40$ ☆ since base 10, you can use log button ☆
$= 1.60$

d) $\ln 3 = 1.099$

over →

Solve using Logs

ex) $25 = 10^{x-1}$ (take log of both sides)

$\log 25 = \boxed{\log_{10} 10}^{x-1}$

$1.39 = x - 1$

$+1 \quad +1$

$x = 2.39$

b) $\log_{10}(3x-2) = 2$ * Change to exponent form *

$10^2 = 3x - 2$

$100 = 3x - 2$

$+2 \quad +2$

$\frac{102}{3} = \frac{3x}{3}$

$x = 34$

c) $e^{x+2} = 8$ * take ln of both sides *

$\ln e^{x+2} = \ln 8$

$x + 2 = -2.08$

$-2 \quad -2$

$x = -4.08$

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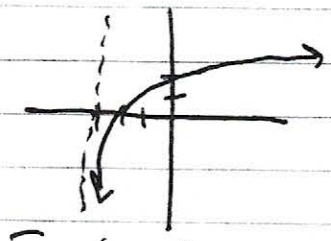
12-24all

6.4 Graph Log Functions

✱ Use Graphing Calculators ✱

ex) Graph $g(x) = \log_2(x+3)$ (use change of base)

$$\frac{\log(x+3)}{\log(2)}$$



b) Find domain and range

$d = (-3, \infty)$

$r = (-\infty, \infty)$

c) Intercepts $(0,2) = y$ intercept, $(-2,0) = x$ intercept

d) asymptote $x = -3$

Finding Inverse ✱ switch x and y ✱

ex) $F(x) = 10^{x+1}$

$y = 10^{x+1}$

$x = 10^{y+1}$ (log both sides)

$\log x = \boxed{\log 10}^{y+1}$

$\log x = y+1$

$-1 \qquad -1$

$y^{-1} = \log x - 1$

✱ Important pg 324 ✱

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14-35 all

6.5 Properties of Logs

$\log_b m \cdot n = \log_b m + \log_b n$

$\log_b m/n = \log_b m - \log_b n$

$\log_b m^n = n \log_b m$

ex| Expand

a) $\log_5 (a^2 b^7)$ (multiplication

$\log_5 a^2 + \log_5 b^7$ (exponent)

$2 \log_5 a + 7 \log_5 b$

b). $\ln \left(\frac{25}{3}\right)$ (Division)

$\ln 25 - \ln 3$

ex| Write expression as a single log

a) $4 \log_4 m + 3 \log_4 n - \log_4 P$

$\boxed{\log_4 m^4 + \log_4 n^3} - \log_4 P$

$\log_4 (m^4 n^3) - \log_4 P$

$\log_4 \left(\frac{m^4 n^3}{P}\right)$

b) $3 \ln 2 - 2 \ln 5$

$\ln 2^3 - \ln 5^2$

$\ln 8 - \ln 25$

$\ln \left(\frac{8}{25}\right)$

over →

Change of base

$$\log_m n = \frac{\log n}{\log m}$$

ex) $\log_2 7 = \frac{\log 7}{\log 2} = 2.81$

ex) Solve

a) $2^x = 7$ (log both sides)

$\log 2^x = \log 7$

$\frac{x \log 2}{\log 2} = \frac{\log 7}{\log 2}$

$x = 2.81$

b) $3^x = 15$

$\log 3^x = \log 15$

$\frac{x \log 3}{\log 3} = \frac{\log 15}{\log 3}$

$x = 2.46$

p9338
16-34 all

6.6 Exponent and Log Equations

ex) $\left(\frac{1}{2}\right)^{x+7} = 4^{3x}$ (log both sides)

$\log\left(\frac{1}{2}\right)^{x+7} = \log 4^{3x}$

$x+7\left(\log\left(\frac{1}{2}\right)\right) = 3x \log 4$ (simplify log)

$(x+7)(-.301) = 3x(.602)$

$-.301x + -2.107 = 1.806x$

$+.301x \qquad\qquad +.301x$

$-2.107 = 2.107x$

$x = -1$

b) $3^{x+1} = 5^{x}$

$\log 3^{x+1} = \log 5^{x}$

$(x+1)\log 3 = x \log 5$

$(x+1).477 = .699x$

$.477x + .477 = .699x$

$-.477x \qquad -.477x$

$\frac{.477}{.222} = \frac{.222x}{.222}$

$x = 2.149$

c) $\cancel{\ln}(x^2-16) = \cancel{\ln}(6x)$ (ln cancel)

$x^2-16 = 6x$ (set =0)

$x^2-6x-16=0$ (factor)

$(x-8)(x+2)=0$

$x = 8, \cancel{-2}$ ☆ check ☆

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16-21 all
28-31 all

6.7 Geometric Sequence and Series

Geometric Series: sequence with a constant ratio between consecutive terms

n	term
1	4
2	12
3	36
4	108

multiply by 3
multiply by 3
multiply by 3

constant $r=3$

$$a_n = \begin{cases} a_1, & n=1 \\ a_{n-1} \cdot r, & n>1 \end{cases}$$

ex) Is the sequence a geometric sequence?
12, 9.6, 7.68, 6.144
Find if the rate is constant.

$\frac{9.6}{12} = .8$

$\frac{7.68}{9.6} = .8$

$\frac{6.144}{7.68} = .8$

b) write recursive definition

$$a_n = \begin{cases} 12, & n=1 \\ a_{n-1} \cdot .8, & n>1 \end{cases}$$

over →

Find the sum of a Geometric Series

$$S_n = \frac{a_1(1-r^n)}{(1-r)}$$

ex) $\sum_{n=1}^{7} 3\left(\frac{2}{3}\right)^{n-1}$ What is the sum

plug in $n=1$ to find a_1

$$3\left(\frac{2}{3}\right)^{1-1} = 3\left(\frac{2}{3}\right)^{0} = 3(1) = 3 = a_1$$

Find r

$r = 2/3$

Find n

$n = 7$

$$S_n = \frac{3(1-(2/3)^7)}{(1-(2/3))} = 8.47$$